The Application of 2-D Visualization on Multi-Dimensional Data

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Outline

- Why Data Visualization
- Scaling—Common Methods Within Political Science
- Newly Emerging Approach—Contrastive Learning
- Comparison between the Two
- Discussion

Why Visualization

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- To present the information of data graphically:
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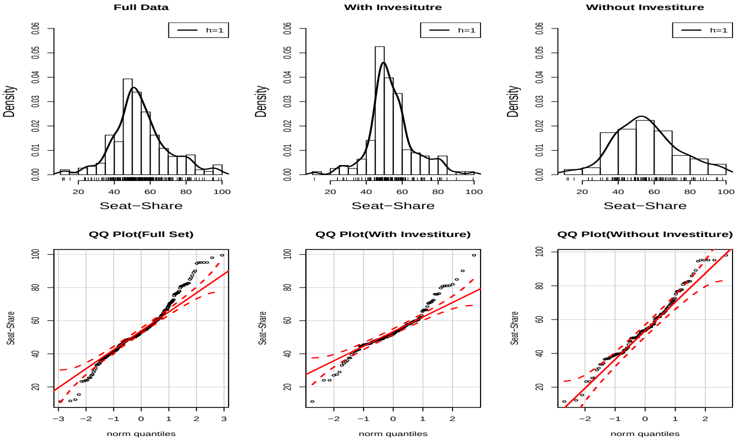
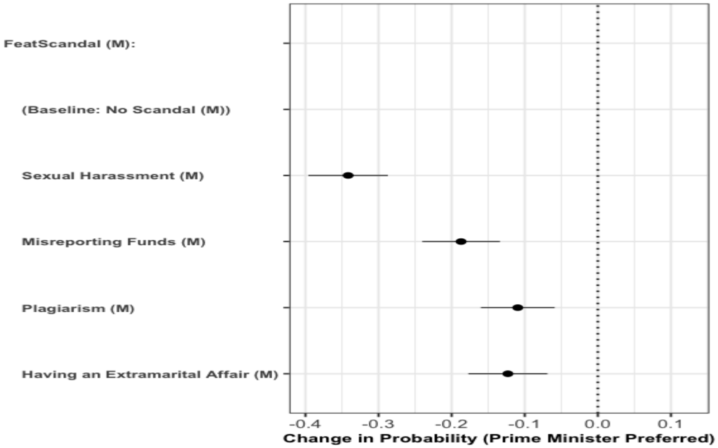
  - Simple
  - Parsimonious

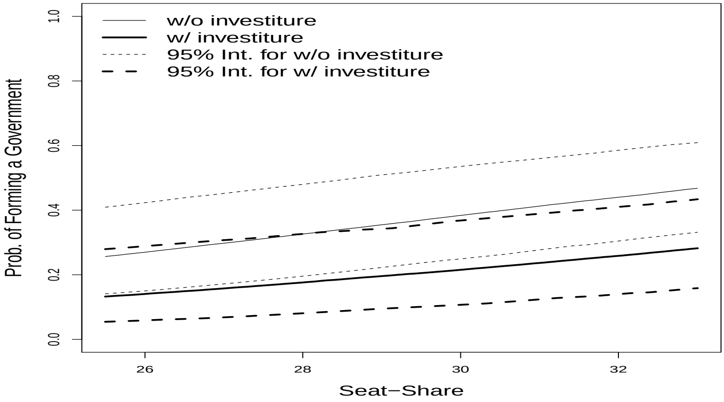
Example—Visualizing Validation of Assumption



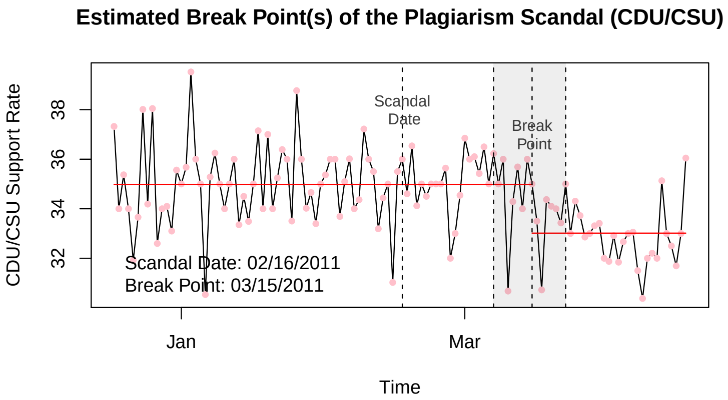
Example—Visualizing Regression Estimation



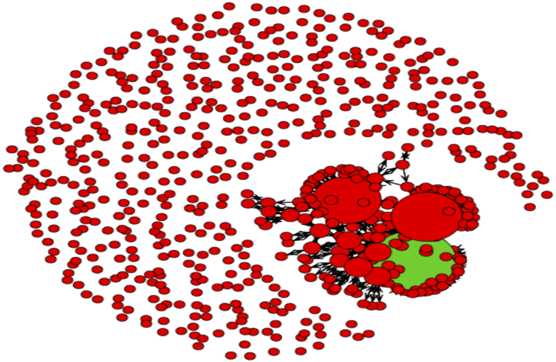
Example—Visualizing Prediction



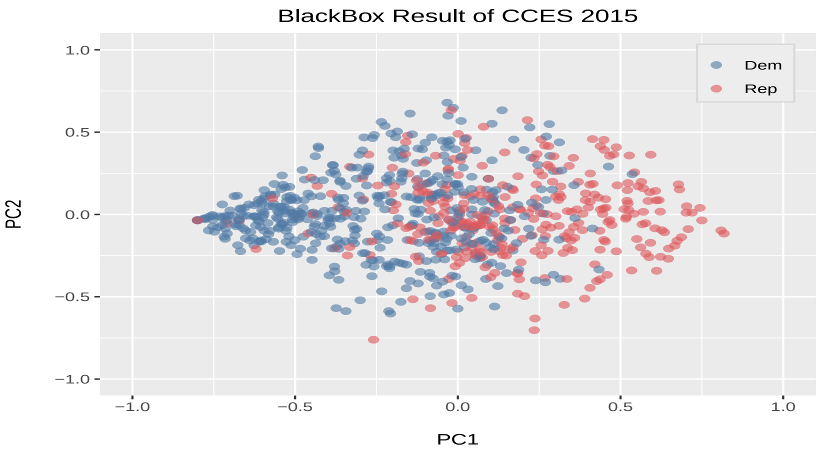
Example—Visualizing Trend



Example—Visualizing Relationships



Example—Visualizing Latent Traits (Today's Focus)



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  - Scaling: Through X to define unknown Y (Unsupervised Learning)

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  - Graphical display: numbers vs. graphics

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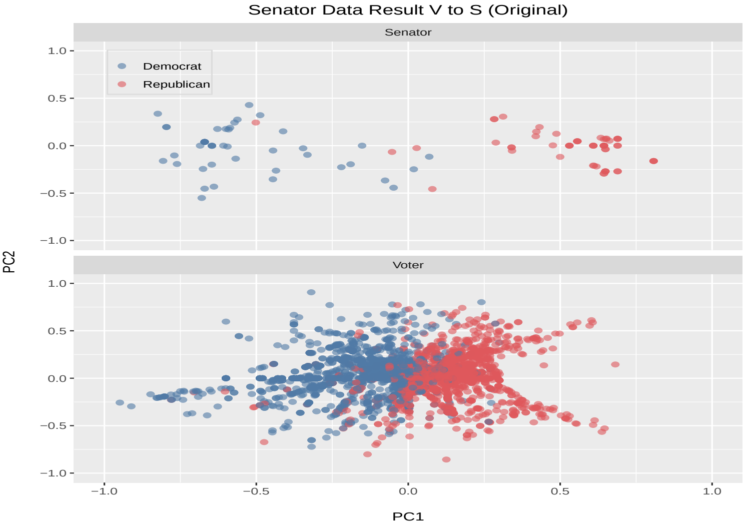
- Scaling methods:
  - Produce a geometric representation of latent trait within the data

Example of Scaling Results



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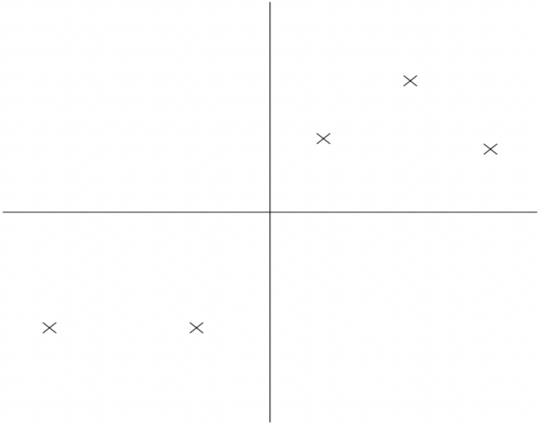
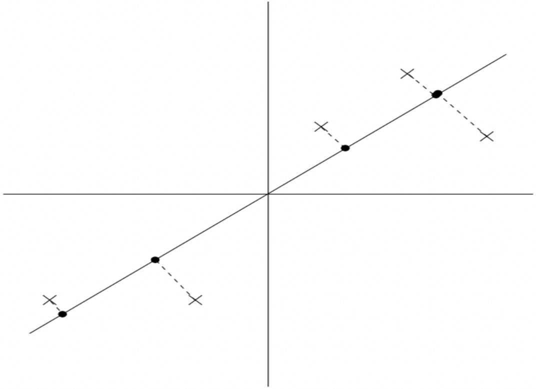
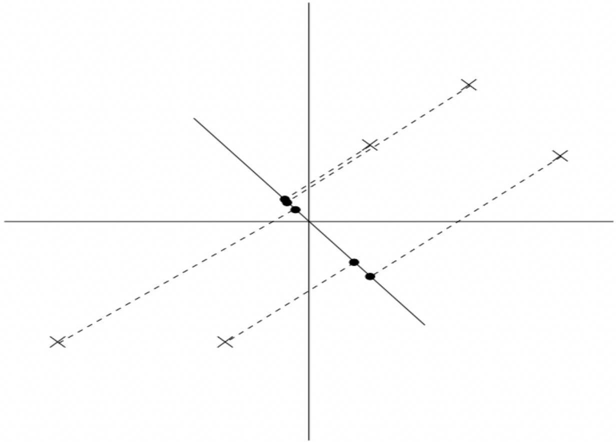


PCA Demonstration II (Andrew Ng, 2015)



PCA Demonstration III (Andrew Ng, 2015)



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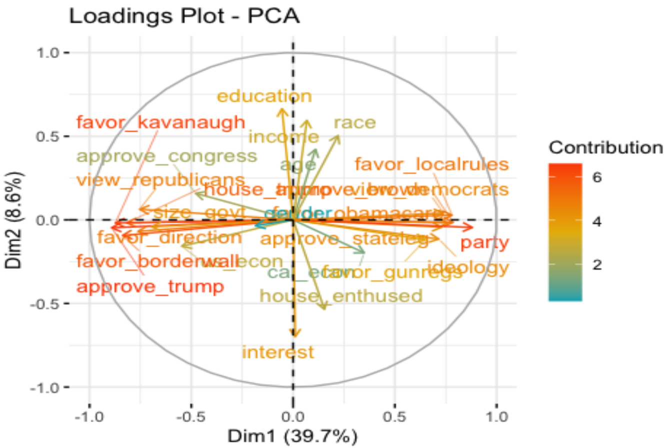
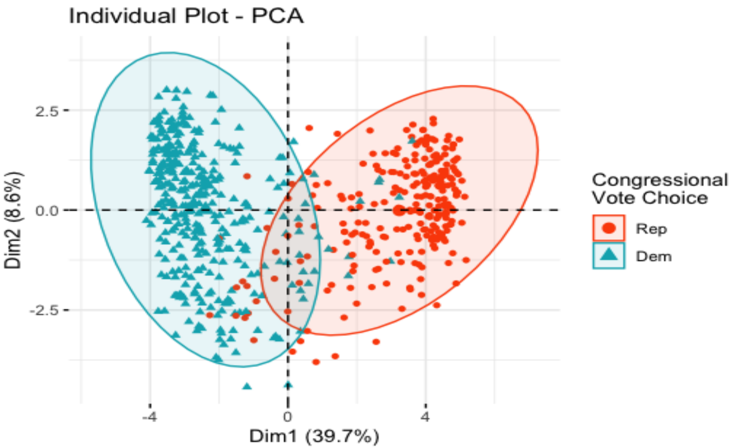
Application of PCA—Loading Plot



Application of PCA—Individual Plot



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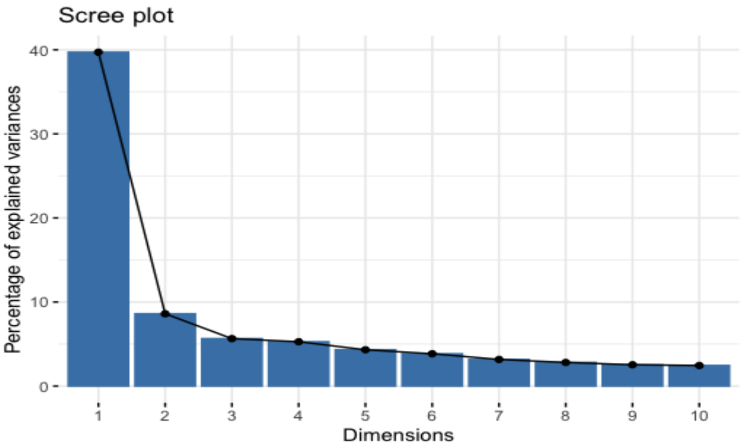
- How to derive principal components:
  - Decompose the variance-covariance matrix of data to find eigenvectors

How Many PCs?

- The elbow method



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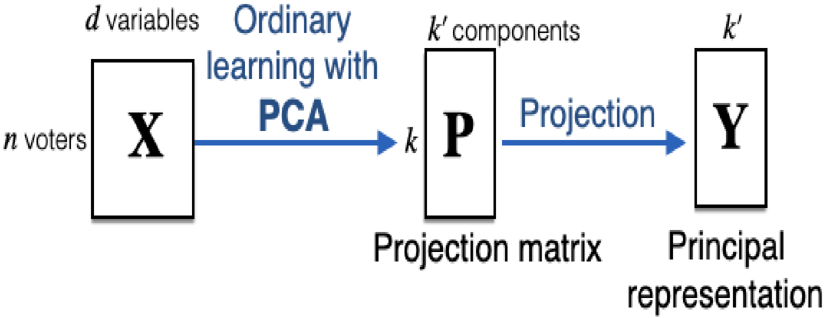
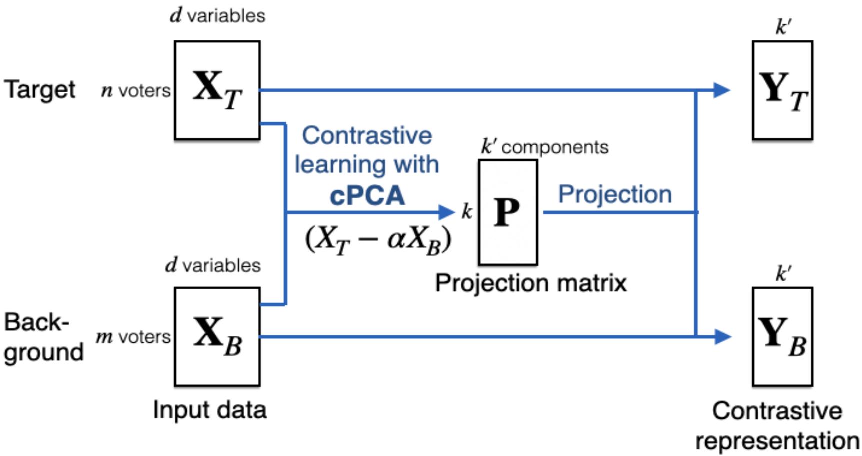
  - Contrastive scaling: Splitting data into different groups first, usually by predefined classes (e.g., party ID), and then compares the data structure of the target group against the background group to find PC(s) on which the target group varies maximally and the background group varies minimally

Ordinary PCA Work Flow



Contrastive PCA Work Flow



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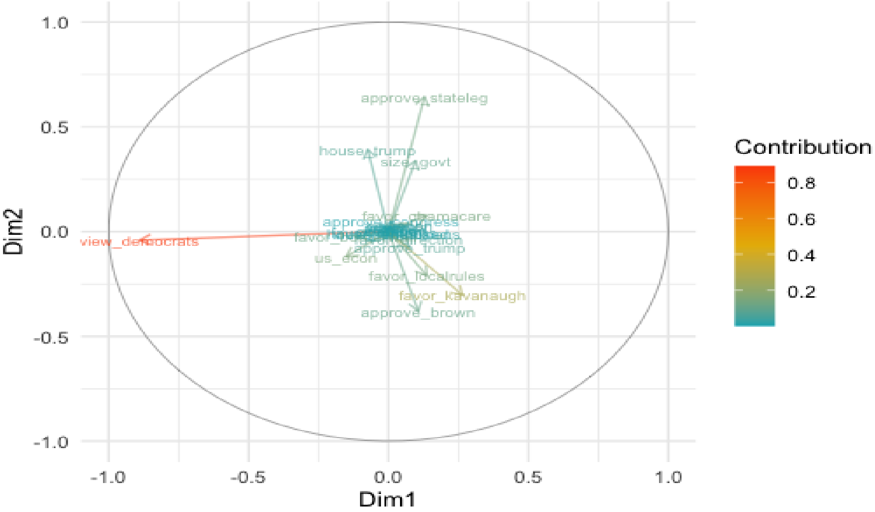
  - One way to intuitively think about α is to treat it as the weight we want to put on the background group—while α is equal to 0, the procedure becomes to apply ordinary scaling on the target group
  - The other way to intuitive think about α is to treat α as a value set—different α means we would like to compare two groups through a different perspective

cPCA Loadings of Californian Voters (TG: Democrats)



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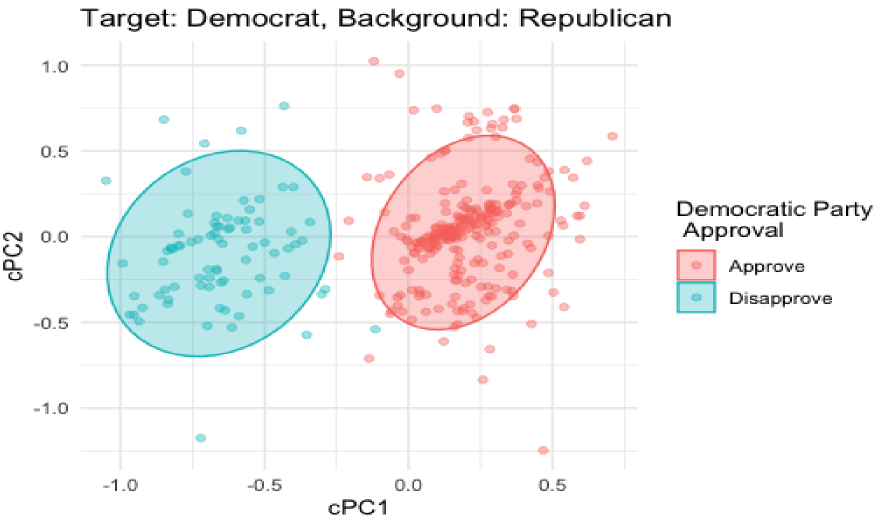
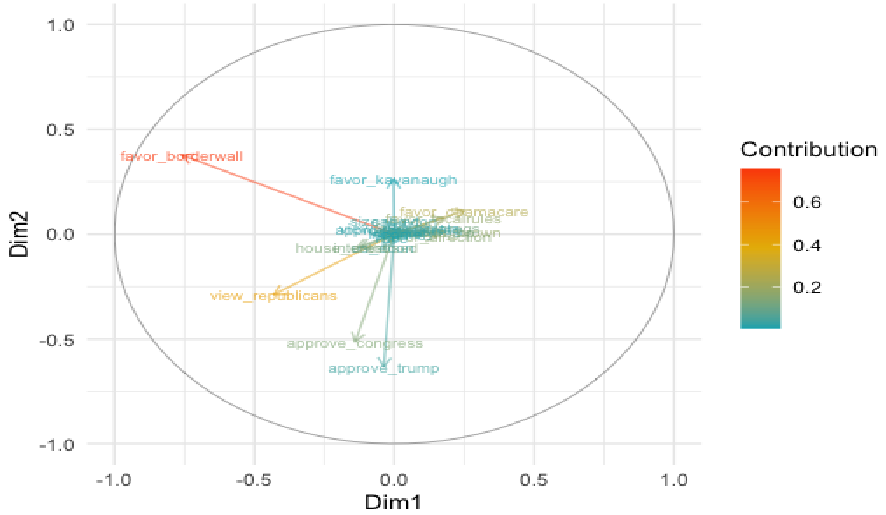
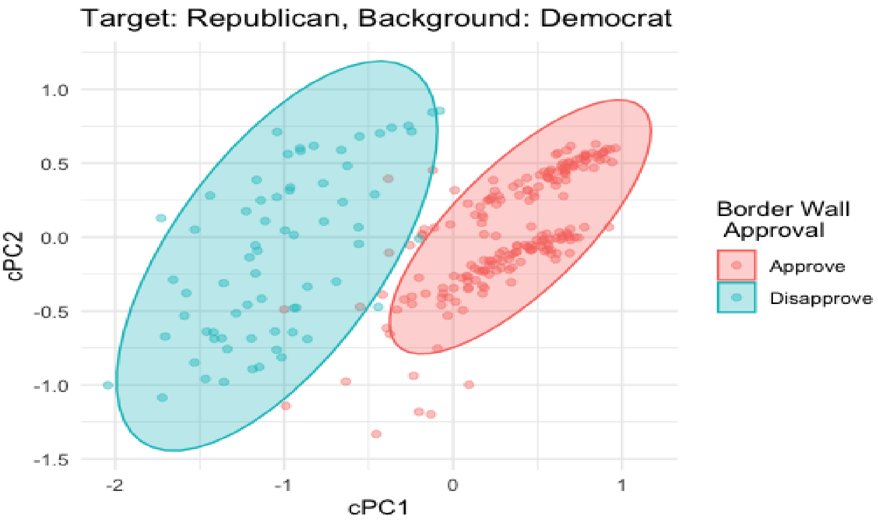


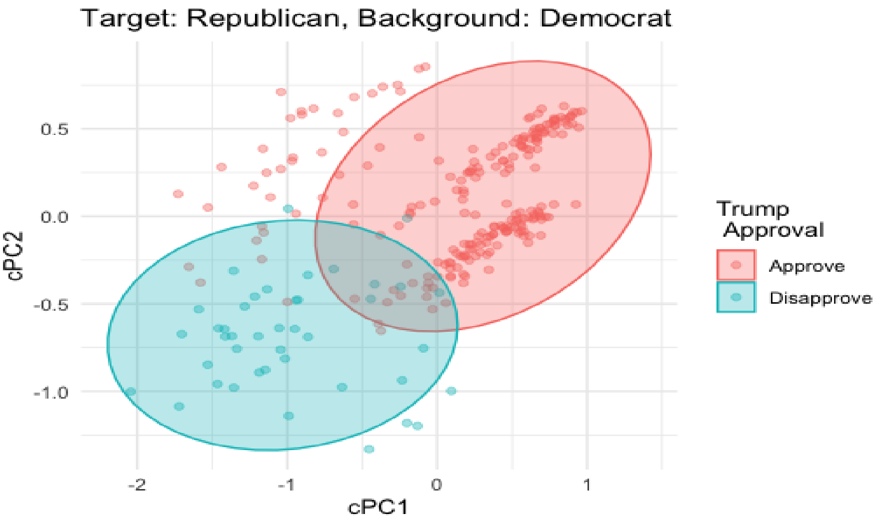
cPCA Loadings of Californian Voters (TG: Republicans)



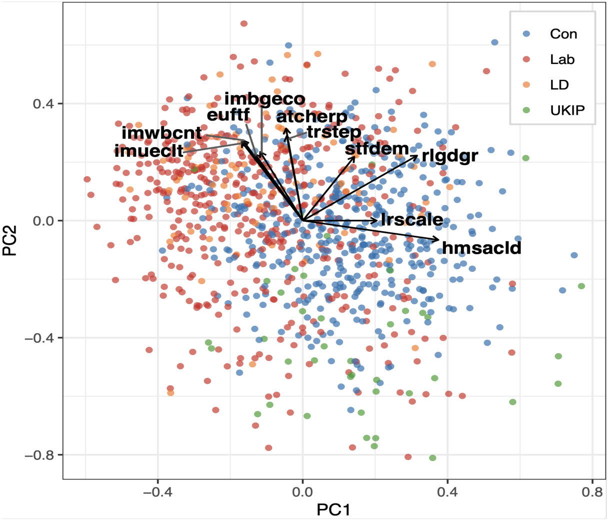
cPCA Individuals of Californian Voters—Wall (TG: Republicans)



cPCA Individuals of Californian Voters—Trump (TG: Republicans)

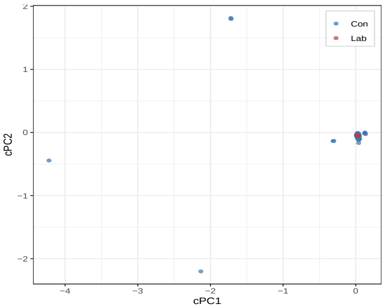


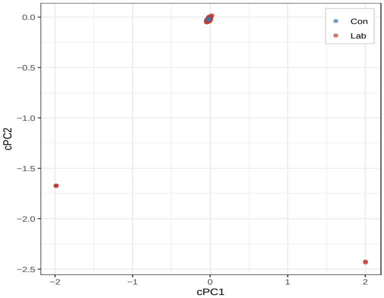
Ordinary Scaling (Black-Box Scaling) Results of ESS 2018 (UK Module)



atcherp: Emotionally Attached to Europe
euftf: European Unification
hmsacld: Gay Adoption
imbgeco: Immigration to Economy
imueclt: Immigration to Culture
imwbcnt: Immigration to Living Quality
lrscale: Left-Right Scale
stfdem: Democracy Satisfaction
trstep: Trust in the European Parliament
(Please refer to Appendix B for details)

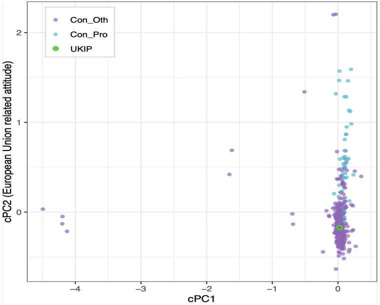
Contrastive MCA Results of ESS 2018 I (UK Module)

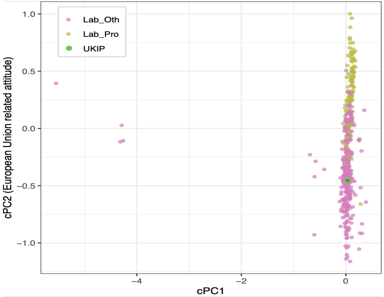




- TG: Conservatives, BG: Labours (Left Panel)
- TG: Labours, BG: Conservatives (Right Panel)

Contrastive MCA Results of ESS 2018 II (UK Module)





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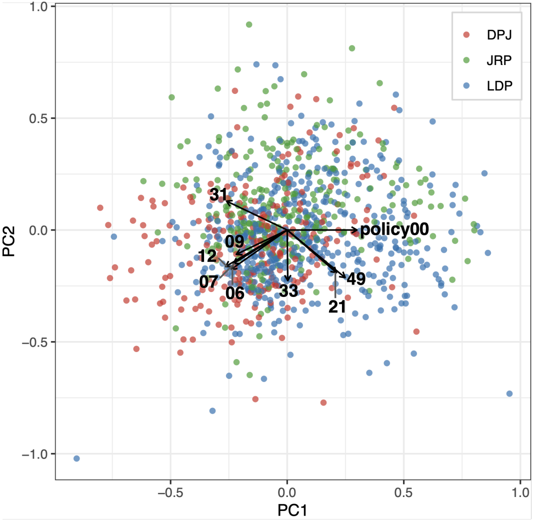
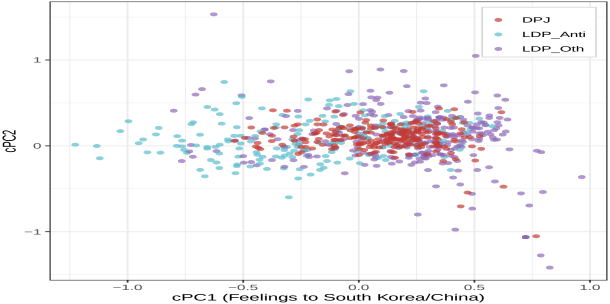


policy00: Ideology
policy06: Constitution Change
policy07: Defense Capacity
policy09: Preemptive Attack
policy12: Collective Defense
policy21: Permanent Foreign Residents with Local Suffrage
policy31: Nuclear Power Plant Restart
policy33: Prime Minister Selection
policy49: Nuclear Power 2030
(Please refer to Appendix B for details)

Contrastive MCA Results of UTAS 2012



- TG: LDP, BG: DPJ

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  - Duck-Mayr and Montegomery (2021) include the level of "dislikeness" in their scaling model which solves the aforementioned issue

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  - Mining data from different perspectives which are overlooked by ordinary approaches
  - Checking similarity between datasets, e.g., balance-checking between the treatment and control groups
  - Providing objective insights for subgroup analysis

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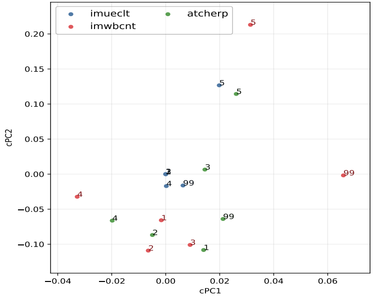
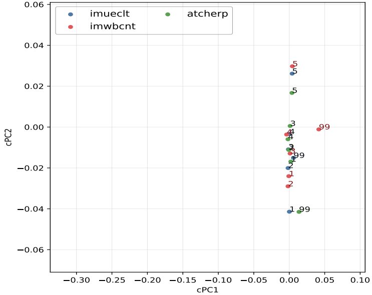
  - Statistical model or machine learning may help only when the design is correct

The End

Thank You!

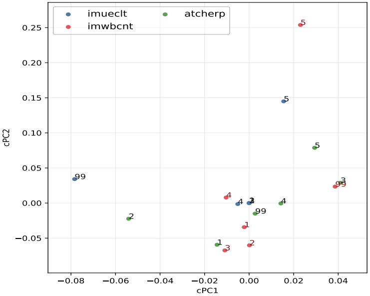
Loadings and Variable Coordinates I

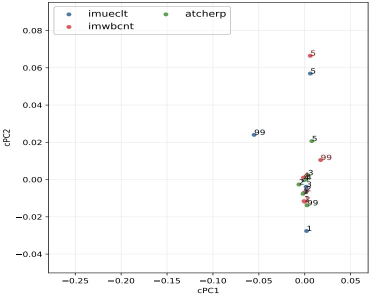




- Loadings for ESS 2018—TG: Labours, BG: UKIP (Left Panel)
- Variable Coordinates for ESS 2018—TG: Labours, BG: UKIP (Right Panel)

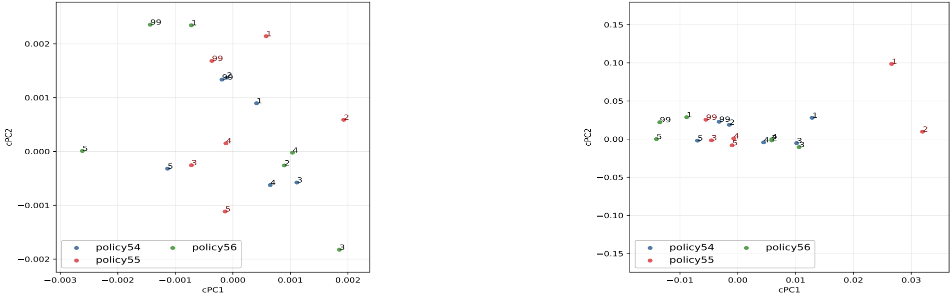
Loadings and Variable Coordinates II





- Loadings for ESS 2018—TG: Conservatives, BG: UKIP (Left Panel)
- Variable Coordinates for ESS 2018—TG: Conservatives, BG: UKIP (Right Panel)

Loadings and Variable Coordinates III



- Loadings for UTAS 2012—TG: LDP, BG: DPJ (Left Panel)
- Variable Coordinates for UTAS 2012—TG: LDP, BG: DPJ (Right Panel)